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Exploring final state hadron structure and SU(3) flavor symmetry breaking effects in $D \to PP$ and $D \to PV$ decays

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Abstract. The non-leptonic two body decays $D \to PP$ and $D \to PV$ are investigated based on the diagrammatic decomposition in a generalized factorization formalism. It is shown that to fit the experimental data, the SU(3) flavor symmetry breaking effects of the coefficients a_is should be considered in $D \to PP$ decay modes. In $D \to PV$ decays, the final state hadron structure due to the pseudoscalar and vector mesons has more important effects on the coefficients a_is than the SU(3) symmetry breaking.

1 Introduction

Charmed meson non-leptonic two body decays have been an interesting subject of research [1–4] for a long time as it can provide useful information on flavor mixing, CP violation [5] and strong interactions. The theoretical settlement of this transition type generally appeals to the factorization hypothesis. Empirically, non-factorizable corrections which result from spectator interactions, final state interactions and resonance effects should be considered. The non-factorizable corrections are believed to be significant [6], and they are relatively hard to be calculated because the charmed quark is not heavy enough to apply the QCD factorization approach [7] or PQCD approach [8] in a reliable manner. Fortunately, a great number of precise experimental data on charmed meson non-leptonic two body decays have been accumulated in recent years. Many new results are expected soon from the dedicated experiments conducted at BES, CLEO, E791, FOCUS, SELEX and the two B factories BaBar and Belle. Phenomenological models based on all kinds of symmetries are quite of importance to guide the theoretical studies and explore new physics [9–11]. But in some cases, the symmetry breaking effects can be significantly enhanced.

In the quark-diagrammatic scenario, all two body non-leptonic weak decays of charmed mesons can be expressed in terms of six distinct quark-graph contributions [1,12]:
(1) a color-favored tree amplitude T,
(2) a color-suppressed tree amplitude C,
(3) a W-exchange amplitude E,
(4) a W-annihilation amplitude A,
(5) a horizontal W-loop amplitude P and
(6) a vertical W-loop amplitude D. The P and D diagrams play little role in practice because the CKM matrix elements have the relation $V_{cs}^* V_{us} \approx -V_{cd}^* V_{ud}$ which will result in cancellations among these diagrams.

Based on SU(3) flavor symmetry, the T, C, E and A amplitudes were fitted from the measured D meson decay modes [10,11]. These amplitudes help one to understand the generality of charmed meson decays. But since SU(3) flavor symmetry breaking effects appear to be important [13,14], these fitted data cannot describe the specific properties in certain decay modes. In [15], we investigated in detail both the Cabibbo-allowed and singly Cabibbo-suppressed $D \to PV$ decays based on the diagrammatic decomposition in the factorization formalism and found that the SU(3) symmetry breaking effects in the $D \to PV$ decays are significant. Two sets of solutions were found in the formalism of factorization. The case (I) solution can provide a satisfactory explanation in a natural manner on the process $D^+ \to \overline{K}^0 K^{*+}$ which is thought to be a puzzle [16]. But the solution is hard to be explained from the theoretical point of view because this solution requires such an unexpectedly large correction from non-factorizable contributions that the strong phase of a_{T_P} has a deviation around 150° from that of the Wilson coefficients c_1. The case (II) solution shows a relatively small correction from non-factorizable contributions and hence seems more reasonable from a theoretical point of view. But the solution predicts a relatively small branching ratio of the process $D^+ \to \overline{K}^0 K^{*+}$ in comparison with the experimental result. With such a treatment via solving fifteen equations for extracting out the same numbers of parameters, it is hard to consider the experimental uncertainties in [15]. To investigate what impacts the experimental uncertainties will have on the extracted parameters, it is useful to make a systematic analysis with taking into account the experimental uncertainties.

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In this paper, we will perform a χ^2 fitting procedure on charmed mesons decaying to a pseudoscalar and a vector meson ($D \to PV$) and also decaying into two light pseudoscalar mesons ($D \to PP$) by using the quark-graph description based on a generalized factorization formalism which reflects SU(3) flavor symmetry breaking effects. Firstly, by dividing these diagrams into factors including SU(3) flavor symmetry breaking effects and introducing parameters describing the overall properties, we arrive at two sets of solutions for the parameters from fitting the experimental data. In the point of view of the diagrammatic decomposition, the generalized QCD parameters $a_i (i = 1, 2)$ will be classified into two sets of parameters a_i^P and a_i^V. The difference between a_i^P and a_i^V arises from the final state hadron structure of the pseudoscalar and vector mesons in $D \to PV$. In $D \to PP$ decays, we will show that, to fit the experimental data, one should classify the parameters a_i into a_i^d and a_i^s, which means that the SU(3) flavor symmetry breaking effects are important and need to be considered in the a_is. Thus we can arrive at the conclusion that the coefficients a_1 and a_2 depend on either the final state hadron structure or on SU(3) flavor symmetry breaking effects. For $D \to PP$ decay modes, the SU(3) flavor symmetry breaking effects play an important role in the coefficients a_1 and a_2, while for $D \to PV$ decays, the final state hadron structure becomes more important for the contributions to the coefficients than the SU(3) symmetry breaking effect does. The contributions of SU(3) flavor symmetry breaking effects to a_1 and a_2 can be neglected in $D \to PV$ decay modes. Using the fitted parameters as inputs, we are led to predictions for the branching ratios of other decay modes which are expected to be measured in the future. In studying the breaking of the SU(3) symmetry relations, we are able to quantify the SU(3) breaking effects. The breaking amount of the SU(3) symmetry relations in some channels can be significant so that it becomes unreliable to use the SU(3) relations to make predictions for some decay modes.

This paper is organized as follows. In Sect. 2, we list the flavor decomposition of the corresponding mesons and present the quark-diagram description for the relevant decay modes. In Sect. 3, the parameterized formalism based on factorization is introduced to investigate the processes. We then perform a fit procedure in Sect. 4 to extract the parameters and present predictions for thirty-three $D \to PP$ decay modes and sixty-two $D \to PV$ decay modes. The SU(3) flavor symmetry breaking effects are discussed in Sect. 5. A short summary and remark is given in the last section.

2 Notation and quark-diagram formalism

We adopt the following quark contents and phase conventions which have been widely used [10–12,17].
(1) Charmed mesons: $D^0 = -c\overline{u}$, $D^+ = c\overline{d}$, $D_s^+ = c\overline{s}$;
(2) Pseudoscalar mesons P: $\pi^+ = u\overline{d}$, $\pi^0 = (d\overline{d} - u\overline{u})/\sqrt{2}$, $\pi^- = -d\overline{u}$, $K^+ = u\overline{s}$, $K^0 = d\overline{s}$, $\overline{K}^0 = s\overline{d}$, $K^- = -s\overline{u}$, $\eta = (-u\overline{u} - d\overline{d} + s\overline{s})/\sqrt{3}$, $\eta' = (u\overline{u} + d\overline{d} + 2s\overline{s})/\sqrt{6}$;
(3) Vector mesons V: $\rho^+ = u\overline{d}$, $\rho^0 = (d\overline{d} - u\overline{u})/\sqrt{2}$, $\rho^- = -d\overline{u}$, $\omega = (u\overline{u} + d\overline{d})/\sqrt{2}$, $K^{*+} = u\overline{s}$, $K^{*0} = d\overline{s}$, $\overline{K}^{*0} = s\overline{d}$, $K^{*-} = -s\overline{u}$, $\phi = s\overline{s}$.

In the above notation, u, d and s quarks transform as a triplet of the flavor SU(3) group, and $-\overline{u}$, $\overline{d}$ and $\overline{s}$ as an antitriplet, so that mesons form isospin multiplets without extra signs. In general, the η–η' mixings are defined as

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix} \tag{1}$$

with $\eta_0 = (u\overline{u}+d\overline{d}+s\overline{s})/\sqrt{3}$ and $\eta_8 = (-u\overline{u}-d\overline{d}+2s\overline{s})/\sqrt{6}$. For convenience, we have taken the mixing parameter as $\phi = 19.5^\circ = \sin^{-1}(1/3)$ which is close to the value $\phi = 15.4^\circ$ extracted from experiment [18].

The partial width Γ for $D \to PP$ and $D \to PV$ decays is expressed in terms of an invariant amplitude $\mathcal{A}$. One has

$$\Gamma(D \to PP) = \frac{p}{8\pi M_D^2}|\mathcal{A}|^2 \tag{2}$$

for $D \to PP$ and

$$\Gamma(D \to PV) = \frac{p^3}{8\pi M_D^2}|\mathcal{A}|^2 \tag{3}$$

for $D \to PV$, where

$$p = \frac{\sqrt{(M_D^2 - (m_1 + m_2)^2)(M_D^2 - (m_1 - m_2)^2)}}{2M_D}$$

denotes the center-of-mass 3-momentum of each final particle.

In $D \to PP$ decays, to describe the flavor SU(3) breaking effects, a subscript s or d is attributed to the T and C diagrams to distinguish the initial c quark transitions to s quark or d quark. The subscript s or d is attached to the diagrams E and A dominated by the weak process $c\overline{q}_1 \to q_2\overline{q}_3$ when the antiquark $\overline{q}_3$ is $\overline{s}$ or $\overline{d}$. In $D \to PV$ decays, a subscript P or V is assigned to T and C, which are induced by $c \to q_3 q_1 \overline{q}_2$ with the spectator quark containing in pseudoscalar or vector final meson. The subscript P or V is labelled to the E and A graphs which are dominated by the weak process $c\overline{q}_1 \to q_2\overline{q}_3$ when the final antiquark $\overline{q}_3$ stays in the pseudoscalar or vector meson. S is added before E or A to distinguish the exchange or annihilation graph involved in final singlet state contributions which result from disconnected graphs. The total contributions of the SE and SA graphs involved in π^0 and ρ^0 mesons are equal to zero because their contributions resulting from $u\overline{u}$ and $-d\overline{d}$ offset each other due to the isospin SU(2) symmetry. In the numerical analysis, we will assume that the contributions of the SE_P and SE_V graphs involved in ω and ϕ mesons are negligibly small since they seem not to contradict with the Okubo–Zweig–Iizuka rule. But the amplitude SA_V seems to play an important role in the $D_s^+ \to \rho^+\eta$ and $D_s^+ \to \rho^+\eta'$ processes [19]. In the ideal mixing case, the process $D_s^+ \to \pi^+\omega$ has the amplitude representation $\frac{1}{\sqrt{2}}(A_V + A_P + 2SA_P)$. Since ω has a similar quark structure in comparison with η and

η', we assume that SA_P has an important contribution in $D_s^+ \to \pi^+\omega$. In the present paper, we shall not consider the processes which receive contributions from SA_V and SA_P diagrams resulting from the final state particles η, η' or ω. The sign flips in the presentation of some relevant Cabibbo-favored modes, as well as that of some doubly Cabibbo-suppressed modes, come from the quark contents of the final light mesons. In the singly Cabibbo-suppressed modes, the sign flips may come either from the quark contents of the final light mesons or from the CKM matrix element $V_{cd}^* V_{ud}$ since $V_{cs}^* V_{us} \approx -V_{cd}^* V_{ud}$ and we choose $V_{cs}^* V_{us}$ in the calculations. In Tables 4 and 5, a prime and double prime are added to the diagrams of singly Cabibbo-suppressed modes and doubly Cabibbo-suppressed modes respectively to distinguish them from the Cabibbo-favored ones.

3 Flavor SU(3) symmetry breaking description in generalized factorization formalism

To investigate the SU(3) flavor symmetry breaking effects, we take the formalism of a generalized factorization approach [2,20].

For $D \to PP$ decays, the amplitudes can be written in the form

$$T_{s,d} = \frac{G_{\rm F}}{\sqrt{2}} V_{q_1 q_2} V_{cq_3}^* a_{T_{s,d}} f_{P_1} (m_{D_i}^2 - m_{P_2}^2) F_0^{D_i \to P_2}(m_{P_1}^2), \quad (4)$$

$$C_{s,d} = \frac{G_{\rm F}}{\sqrt{2}} V_{q_1 q_2} V_{cq_3}^* a_{C_{s,d}} f_{P_1} (m_{D_i}^2 - m_{P_2}^2) F_0^{D_i \to P_2}(m_{P_1}^2), \quad (5)$$

$$E_{s,d} = \frac{G_{\rm F}}{\sqrt{2}} V_{q_1 q_3} V_{cq_2}^* a_{E_{s,d}} f_{D_i}, \quad (6)$$

$$A_{s,d} = \frac{G_{\rm F}}{\sqrt{2}} V_{q_2 q_3} V_{cq_1}^* a_{A_{s,d}} f_{D_i}. \quad (7)$$

For $D \to PV$ decays, the amplitudes can be written in the form

$$T_V = \frac{G_{\rm F}}{\sqrt{2}} V_{q_1 q_2} V_{cq_3}^* a_{T_V} 2 f_P m_{D_i} A_0^{D_i \to V}(m_P^2), \quad (8)$$

$$T_P = \frac{G_{\rm F}}{\sqrt{2}} V_{q_1 q_2} V_{cq_3}^* a_{T_P} 2 f_V m_{D_i} F_1^{D_i \to P}(m_V^2), \quad (9)$$

$$C_V = \frac{G_{\rm F}}{\sqrt{2}} V_{q_1 q_2} V_{cq_3}^* a_{C_V} 2 f_P m_{D_i} A_0^{D_i \to V}(m_P^2), \quad (10)$$

$$C_P = \frac{G_{\rm F}}{\sqrt{2}} V_{q_1 q_2} V_{cq_3}^* a_{C_P} 2 f_V m_{D_i} F_1^{D_i \to P}(m_V^2), \quad (11)$$

$$E_{V,P} = \frac{G_{\rm F}}{\sqrt{2}} V_{q_1 q_3} V_{cq_2}^* a_{E_{V,P}} 2 f_{D_i} m_{D_i}, \quad (12)$$

$$A_{V,P} = \frac{G_{\rm F}}{\sqrt{2}} V_{q_2 q_3} V_{cq_1}^* a_{A_{V,P}} 2 f_{D_i} m_{D_i}. \quad (13)$$

D_i denotes $D^\pm$, D_0 or D_s. F_0, F_1 and A_0 are formfactors defined in the following formalism:

$$\begin{aligned} &\langle P(p)|\bar{q}\gamma^\mu c|D(p_D)\rangle \\ &= \left[(p_D + p)_\mu - \frac{m_D^2 - m_P^2}{q^2} q^\mu\right] F_1(q^2) \\ &\quad + \frac{m_D^2 - m_P^2}{q^2} q^\mu F_0(q^2), \end{aligned} \quad (14)$$

$$\begin{aligned} &\langle V(p)|\bar{q}\gamma^\mu(1-\gamma^5)c|D(p_D)\rangle \\ &= -\mathrm{i}(m_D + m_V) A_1(q^2) \left(\epsilon^{*\mu} - \frac{\epsilon^* \cdot q}{q^2} q^\mu\right) \\ &\quad + \mathrm{i}\frac{A_2(q^2)}{m_D + m_V} (\epsilon^* \cdot q) \left((p_D + p)^\mu - \frac{m_D^2 - m_V^2}{q^2} q^\mu\right) \\ &\quad - \mathrm{i}\frac{2 m_V}{q^2} (\epsilon^* \cdot q) A_0(q^2) q^\mu - \frac{2V(q^2)}{m_D + m_V} \epsilon^{\mu\alpha\beta\gamma} \epsilon_\alpha^* p_{D\beta} p_\gamma, \end{aligned} \quad (15)$$

with $q = p_D - p$. f_P and f_V are decay constants defined by

$$\langle P(p)|\bar{q}_1 \gamma^\mu \gamma_5 q_2|0\rangle = -i f_P p^\mu, \quad (16)$$

$$\langle V(p)|\bar{q}_1 \gamma^\mu q_2|0\rangle = f_V m_V \epsilon^\mu. \quad (17)$$

In the naive factorization hypothesis, one has the following equalities:

$$a_{T_s} = a_{T_d} = a_{T_V} = a_{T_P} = a_1(\mu), \quad (18)$$

$$a_{C_s} = a_{C_d} = a_{C_V} = a_{C_P} = a_2(\mu), \quad (19)$$

with

$$a_1(\mu) = c_1(\mu) + \frac{1}{N_c} c_2(\mu), \quad (20)$$

$$a_2(\mu) = c_2(\mu) + \frac{1}{N_c} c_1(\mu), \quad (21)$$

denoting the relations between the quantities $a_{1,2}$ and Wilson coefficients $c_{1,2}$. N_c is the number of colors. μ is the renormalization scale at which c_1 and c_2 are evaluated. So a_1 and a_2 are common real quantities of a certain process on the quark level. To be more explicit, for decay modes induced by the $c \to s$ transition, a_1 and a_2 are invariant among all modes in a naive factorization hypothesis.

However, the naive factorization approach meets difficulties in describing all charmed meson decays, particularly for the decay modes which are involved in the color-suppressed diagrams due to the smallness of $|a_2|$. Furthermore, the coefficients a_1 and a_2 in (18) and (19) depend on the renormalization scale and the γ_5 scheme at the next to leading order expansion. It is necessary to consider non-factorizable corrections due to hard spectator interactions, final state interactions and resonance effects etc. For illustration purposes, we take an simple example in which a_1 and a_2 have the following form:

$$a_1(\mu) = c_1(\mu) + \left(\frac{1}{N_c} + \chi_1(\mu)\right) c_2(\mu), \quad (22)$$

$$a_2(\mu) = c_2(\mu) + \left(\frac{1}{N_c} + \chi_2(\mu)\right) c_1(\mu), \quad (23)$$

with $\chi_1(\mu)$ and $\chi_2(\mu)$ terms partially denoting the non-factorizable corrections. With these corrections the equalities (18) and (19) are not yet satisfied because each a_i

should contain terms from different corrections. The corrections can also bring about phase differences among these coefficients, and then a_is ($i = T_{s,d,V,P}, C_{s,d,V,P}$) become complex. Currently, explicit calculations of the total corrections are not yet possible. In $D \to PV$ decays, we shall take all a_is as independent complex parameters and assume that the corrections do not depend on individual decay process at certain scale. In other words, we do not consider SU(3) flavor symmetry violation contributions to a_is and it is supposed that mass factors, decay constants and formfactors have taken on the whole SU(3) symmetry breaking effects, while in $D \to PP$ decays, the mass factors, the formfactors and decay constants fail to account for the large SU(3) flavor symmetry breaking effects in $D \to \pi\pi$, $D \to \pi K$ and $D \to KK$. Contributions from the corrections to naive factorization may cause large SU(3) symmetry breaking [14]. Two sets of coefficients a_i^s and a_i^d are introduced to describe the SU(3) flavor symmetry breaking effects induced by the corrections. In both $D \to PP$ and $D \to PV$ decays, the SU(3) symmetry breaking effects are not considered in the strong phases in our present analysis.

The exchange and annihilation diagrams have the following expressions in the naive factorization approach:

$$E_{s,d} = \frac{G_F}{\sqrt{2}} V_{q_1q_3} V^*_{cq_2} a^{nf}_{E_{s,d}} f_{D_i} (m^2_{P_1} - m^2_{P_2}) F_0^{P_1P_2}(m^2_{D_i}), \tag{24}$$

$$A_{s,d} = \frac{G_F}{\sqrt{2}} V_{q_2q_3} V^*_{cq_1} a^{nf}_{A_{s,d}} f_{D_i} (m^2_{P_1} - m^2_{P_2}) F_0^{P_1P_2}(m^2_{D_i}), \tag{25}$$

$$E_{V,P} = \frac{G_F}{\sqrt{2}} V_{q_1q_3} V^*_{cq_2} a^{nf}_{E_{V,P}} 2 f_{D_i} m_{D_i} A_0^{PV}(m^2_{D_i}), \tag{26}$$

$$A_{V,P} = \frac{G_F}{\sqrt{2}} V_{q_2q_3} V^*_{cq_1} a^{nf}_{A_{V,P}} 2 f_{D_i} m_{D_i} A_0^{PV}(m^2_{D_i}). \tag{27}$$

The formfactors $F_0^{P_1P_2}(m^2_{D_i})$ and $A_0^{PV}(m^2_{D_i})$ involved in the above formula are not manifestly relating directly to experimental measurements. The factorizable contributions of the exchange and annihilation diagrams are believed to be small. Through intermediate states, these diagrams relate to the tree diagram T and color-suppressed diagram C [21,22]. Their contributions may be important and cannot be ignored. In our present considerations, we use a_{E_i,A_i} ($i = s, d, V, P$) defined in (6), (7), (12) and (13) as global parameters to describe mainly the contributions of intermediate states. By these definitions, the parameters a_{E_i,A_i} will have two dimensions of energy in $D \to PP$ and will be dimensionless in $D \to PV$.

4 Numerical analysis and results

The explicit evaluation of the relevant formfactors in the factorization formula (4), (5) and (8)–(11) is a hard task because of the non-perturbative long distance effects of QCD. Various methods, such as QCD sum rules [23, 24], lattice simulations [25,26] and the phenomenological quark model [27,28], have been developed to estimate the long distance effects to rather high certainties. The formfactors of D mesons decaying to light mesons have been widely discussed in [29–34]. In our present considerations, we shall use the results of formfactors obtained by Bauer, Stech and Wirbel [2,29] based on the quark model. They have been found to be rather successful in describing a number of processes concerning D mesons. The values of the relevant formfactors evaluated at $q^2 = 0$ are listed in Table 1. For the dependence on q^2, the formfactors are assumed to behave as a monopole dominance:

$$D \to P: \quad F_0(q^2) = \frac{F_0(0)}{1 - q^2/m^2_{F^{**}}}, \tag{28}$$

$$F_1(q^2) = \frac{F_1(0)}{1 - q^2/m^2_{F^*}}, \tag{29}$$

$$D \to V: \quad A_0(q^2) = \frac{A_0(0)}{1 - q^2/m^2_F}, \tag{30}$$

where m_F, m_{F^*} and $m_{F^{**}}$ are the pole masses given in Table 1.

It is noted that it is more appropriate view the formfactors as the relative scaling factors that characterize one source of SU(3) flavor symmetry breaking effects in hadronic matrix elements since we take the a_is as free parameters that need to be extracted from experimental inputs in the present method. The relative ratio between the formfactors is what we really care about.

The input values for the light pseudoscalar and vector decay constants are presented in Table 2 [35,36]. These values generally coincide with experiments. The decay constants f^u_η, f^s_η, $f^u_{\eta'}$ and $f^s_{\eta'}$ involved in the factorization formula should be defined as follows [35]:

$$\langle 0|\bar{u}\gamma^\mu\gamma_5 u|\eta^{(\prime)}(p)\rangle = \mathrm{i} f^u_{\eta^{(\prime)}} p^\mu, \tag{31}$$

$$\langle 0|\bar{s}\gamma^\mu\gamma_5 s|\eta^{(\prime)}(p)\rangle = \mathrm{i} f^s_{\eta^{(\prime)}} p^\mu. \tag{32}$$

Then the quantities f^u_η, f^s_η, $f^u_{\eta'}$ and $f^s_{\eta'}$ take in the formalism the form

$$f^u_\eta = \frac{f_8}{\sqrt{6}}\cos\phi + \frac{f_0}{\sqrt{3}}\sin\phi, \tag{33}$$

$$f^s_\eta = \frac{2f_8}{\sqrt{6}}\cos\phi - \frac{f_0}{\sqrt{3}}\sin\phi, \tag{34}$$

$$f^u_{\eta'} = -\frac{f_8}{\sqrt{6}}\sin\phi + \frac{f_0}{\sqrt{3}}\cos\phi, \tag{35}$$

$$f^s_{\eta'} = \frac{2f_8}{\sqrt{6}}\sin\phi + \frac{f_0}{\sqrt{3}}\cos\phi. \tag{36}$$

Making use of these definitions, the following factorization formulas are adopted in the $D \to \eta(\eta')V$ transition calculation:

$$2C_V(D_i \to \eta V) \tag{37}$$
$$= \frac{G_F}{\sqrt{2}} V_{q_1q_2} V^*_{cq_3} a_{C_V} 2(f^u_\eta + f^s_\eta) m_{D_i} A_0^{D_i\to V}(m^2_\eta),$$

$$C_V(D_i \to \eta' V) \tag{38}$$

Table 1. Relevant formfactors at zero momentum transfer for $D \to P$ and $D \to V$ transitions and pole masses in BSW model

Decay	$D \to \pi$	$D \to \rho(\omega)$	$D \to K$	$D \to K^*$	$D_s \to K$	$D_s \to K^*$	$D_s \to \phi$	$D \to \eta/\eta'$	$D_s \to \eta/\eta'$
F_1	0.692		0.762		0.643			0.681/0.655	0.723/0.704
A_0		0.669		0.733		0.634	0.700		
m_F (GeV)		1.87		1.97		1.87	1.97		
m_{F^*} (GeV)	2.01		2.11		2.01			2.01	2.11
$m_{F^{**}}$ (GeV)	2.47		2.60		2.47			2.47	2.60

Table 2. Values of decay constants in MeV

f_π	f_K	f_8	f_0	f_D	f_{D_s}	f_ρ	f_{K^*}	f_ω	f_ϕ	f_{D^*}	$f_{D_s^*}$
134	158	168	157	200	234	210	214	195	233	230	275

Table 3. Parameters a_is fitted from experimental data at 1σ errors. The first entry is for amplitude and the second entry for the strong phase. $a_1^{s,d,V,P}$ and $a_2^{s,d,V,P}$ denote $a_{T_{s,d,V,P}}$ and $a_{C_{s,d,V,P}}$ respectively

	$D \to PP$			$D \to PV$	
	FIT α	FIT β		FIT A	FIT B
χ^2/d.o.f.	4.06/4	8.16/4		8.22/7	10.30/7
a_1^s	1.08 ± 0.04	1.10 ± 0.03	a_1^V	1.13 ± 0.08	1.10 ± 0.07
	–	–		–	–
a_1^d	1.04 ± 0.09	1.09 ± 0.09	a_1^P	1.29 ± 0.04	1.29 ± 0.04
	$(8.73 \pm 7.96)^\circ$	$(11.98 \pm 7.85)^\circ$		$(10.04 \pm 16.62)^\circ$	$(-1.36 \pm 13.52)^\circ$
a_2^s	-0.73 ± 0.04	-0.73 ± 0.04	a_2^V	-1.19 ± 0.06	-1.00 ± 0.05
	$(-26.76 \pm 1.60)^\circ$	$(-26.25 \pm 1.55)^\circ$		$(-11.09 \pm 20.01)^\circ$	$(-10.74 \pm 10.31)^\circ$
a_2^d	-0.36 ± 0.20	-0.65 ± 0.06	a_2^P	-0.78 ± 0.03	-0.77 ± 0.02
	$(-53.40 \pm 28.65)^\circ$	$(-35.12 \pm 14.50)^\circ$		$(-21.75 \pm 1.38)^\circ$	$(-22.15 \pm 2.23)^\circ$
a_E^s (GeV^2)	0.24 ± 0.11	0.25 ± 0.11	a_E^V	0.07 ± 0.03	0.26 ± 0.04
	$(-49.43 \pm 20.63)^\circ$	$(-58.75 \pm 19.48)^\circ$		$(-166.87 \pm 50.96)^\circ$	$(-115.50 \pm 12.78)^\circ$
a_E^d (GeV^2)	1.01 ± 0.08	1.01 ± 0.08	a_E^P	0.51 ± 0.02	0.50 ± 0.03
	$(-120.94 \pm 2.92)^\circ$	$(-122.02 \pm 2.82)^\circ$		$(82.82 \pm 4.01)^\circ$	$(78.55 \pm 5.73)^\circ$
a_A^s (GeV^2)	–	–	a_A^V	0.52 ± 0.05	0.53 ± 0.05
	–	–		$(-76.66 \pm 25.78)^\circ$	$(-78.24 \pm 26.13)^\circ$
a_A^d (GeV^2)	0.43 ± 0.09	0.46 ± 0.09	a_A^P	0.59 ± 0.03	0.60 ± 0.02
	$(90.04 \pm 13.75)^\circ$	$(89.94 \pm 14.27)^\circ$		$(-76.29 \pm 41.25)^\circ$	$(-77.93 \pm 31.52)^\circ$

$$= \frac{G_F}{\sqrt{2}} V_{q_1 q_2} V_{cq_3}^* a_{C_V} 2(f_{\eta'}^s - f_{\eta'}^u) m_{D_i} A_0^{D_i \to V}(m_{\eta'}^2).$$

The other parameters used in the numerical calculation are the masses of relevant mesons, the lifetimes of charmed mesons and the relevant CKM matrix elements. We adopt the relevant values given in [37].

For convenience, we may express the complex parameters a_i by

$$a_i = |a_i| e^{i\delta_{a_i}}. \qquad (39)$$

The δ_{a_i}s characterize the strong phases. One can always choose $\delta_{a_{T_s}} = 0$ in $D \to PP$ and $\delta_{a_{T_V}} = 0$ in $D \to PV$ so that all the other strong phases are relative to $\delta_{a_{T_s}}$ and $\delta_{a_{T_V}}$. There are 15 independent parameters to be extracted from experiment in both $D \to PP$ and $D \to PV$.

To conduct a fit procedure, we construct a χ^2 function which has the following form:

$$\chi^2 = \sum_j \frac{(f_j(a_i) - \langle f_j \rangle)^2}{\sigma_j^2}, \qquad (40)$$

where $\langle f_j \rangle$ and σ_j are the central values and the corresponding errors of the experimentally measured observables. $f_j(a_i)$ are the theoretical expressions for the observables. They are functions of the parameters a_i. The set of a_is which minimizes the χ^2 function will be regarded as the best estimated values.

Due to the limited number of data points, we shall neglect the SU(3) symmetry breaking in the annihilation diagrams in $D \to PP$ decays and take $a_A^s = a_A^d$ in the fit to make predictions for $D^+ \to K^0\pi^+$, $D^+ \to K^+\pi^0$,

Table 4. Predicted branching ratios for charmed mesons decaying to two pseudoscalar mesons. Single prime and double primes are added to the representations to denote the singly Cabibbo-suppressed processes and doubly Cabibbo-suppressed processes. C_{s_1} and C_{s_2}, as well as C_{d_1} and C_{d_2}, result from the exchange of the final mesons

	Decay modes	Representation	Experimental $\mathcal{B} \times 10^{-2}$	Present $\mathcal{B} \times 10^{-2}$ FIT α	FIT β	LP[40] $\mathcal{B} \times 10^{-2}$
D^0	$K^-\pi^+$	$T_s + E_d$	3.80 ± 0.09	3.79	3.80	3.847
	$\overline{K}^0\pi^0$	$\frac{1}{\sqrt{2}}(C_s - E_d)$	2.28 ± 0.22	2.27	2.24	1.310
	$\overline{K}^0\eta$	$\frac{1}{\sqrt{3}}C_s$	0.76 ± 0.11	0.80	0.81	
	$\overline{K}^0\eta'$	$-\frac{1}{\sqrt{6}}(C_s + 3E_d)$	1.87 ± 0.28	1.85	1.88	
	$\pi^+\pi^-$	$-(T'_d + E'_d)$	0.143 ± 0.007	0.144	0.144	0.151
	$\pi^0\pi^0$	$-\frac{1}{\sqrt{2}}(C'_d - E'_d)$	0.084 ± 0.022	0.078	0.097	0.115
	K^+K^-	$T'_s + E'_s$	0.412 ± 0.014	0.413	0.413	0.424
	$K^0\overline{K}^0$	$E'_s - E'_d$	0.071 ± 0.019	0.069	0.062	0.130
	$K^+\pi^-$	$-(T''_d + E''_s)$	0.0148 ± 0.0021	0.0150	0.0151	0.033
	$\eta\pi^0$	$\frac{1}{\sqrt{6}}(C'_s + C'_{d_1} - C'_{d_2} - 2E'_d - SE')$	–	0.069	0.068	
	$\eta'\pi^0$	$\frac{1}{\sqrt{12}}(2C'_s - C'_{d_1} + C'_{d_2} + 2E'_d + 4SE')$	–	0.088	0.091	
	$\eta\eta$	$\frac{1}{3\sqrt{2}}(2C'_s + 2C'_d - 2E'_s + 2E'_d + 4SE)$	–	0.011	0.016	
	$\eta\eta'$	$\frac{1}{\sqrt{18}}(2C'_{s_1} - C'_{s_2} - C'_{d_1} - C'_{d_2} - 4E'_s - 2E'_d - 7SE)$	–	0.026	0.030	
	$K^0\pi^0$	$-\frac{1}{\sqrt{2}}(C''_d - E''_s)$	–	0.002	0.005	0.008
	$K^0\eta$	$-\frac{1}{\sqrt{3}}(C''_d - E''_s + SE'')$	–	0.001	0.002	
	$K^0\eta'$	$\frac{1}{\sqrt{6}}(C''_d + 3E''_s + 4SE'')$	–	0.0	0.0	
D^+	$\overline{K}^0\pi^+$	$T_s + C_s$	2.77 ± 0.18	2.76	2.76	2.939
	$\pi^+\pi^0$	$-\frac{1}{\sqrt{2}}(T'_d + C'_d)$	0.25 ± 0.07	0.25	0.19	0.185
	$\eta\pi^+$	$\frac{1}{\sqrt{3}}(T'_d + C'_s + C'_d + 2A'_d + SA')$	0.30 ± 0.06	0.34	0.37	
	$\eta'\pi^+$	$-\frac{1}{\sqrt{6}}(T'_d - 2C'_s + C'_d + 2A'_d + 4SA')$	0.50 ± 0.10	0.45	0.42	
	$K^+\overline{K}^0$	$T'_s - A'_d$	0.58 ± 0.06	0.62	0.62	0.764
	$K^0\pi^+$	$-(C''_d + A''_s)$	–	0.012	0.026	0.053
	$K^+\pi^0$	$-\frac{1}{\sqrt{2}}(T''_d - A''_s)$	–	0.021	0.023	0.055
	$K^+\eta$	$\frac{1}{\sqrt{3}}(T''_d + SA'')$	–	0.011	0.012	
	$K^+\eta'$	$-\frac{1}{\sqrt{6}}(T''_d + 3A''_s + 4SA'')$	–	0.005	0.006	
D_s^+	$\overline{K}^0K^+$	$C_s + A_d$	3.6 ± 1.1	3.06	3.13	4.623
	$\pi^+\eta$	$\frac{1}{\sqrt{3}}(T_s - 2A_d - SA)$	1.7 ± 0.5	1.05	1.09	1.131
	$\pi^+\eta'$	$\frac{2}{\sqrt{6}}(T_s + A_d + 2SA)$	3.9 ± 1.0	4.19	4.43	
	π^+K^0	$-(T'_d - A'_s)$	< 0.8	0.24	0.26	0.373
	π^0K^+	$-\frac{1}{\sqrt{2}}(C'_d + A'_s)$	–	0.047	0.090	0.146
	ηK^+	$\frac{1}{\sqrt{3}}(T'_s + C'_s + C'_d - SA')$	–	0.055	0.040	0.300
	$\eta'K^+$	$\frac{1}{\sqrt{6}}(2T'_s + 2C'_s - C'_d + 3A'_s + 4SA')$	–	0.090	0.102	
	K^+K^0	$-(T''_d + C''_d)$	–	0.014	0.010	0.012

$D_s^+ \to \pi^+K^0$ and $D_s^+ \to \pi^0K^+$ etc. The obtained results will provide a reference for further studies. Note that all these modes that are yet to be seen are dominated by tree type diagrams; the SU(3) breaking effects in $a_A^{s(d)}$ are less significant. There are 17 experimental data points for 13 parameters in $D \to PP$ decays and 22 data points for 15 parameters in $D \to PV$ decays, as shown in Tables 4 and 5 respectively. We list in Table 3 the parameters with 1σ errors obtained in our present analysis. FIT α and FIT A are obtained without any constraints to the parameters. A large $|a_2^s/a_2^d| \approx 2.0$ ratio predicted by FIT α is an indication of inscrutably large flavor SU(3) breaking effects. Constraining the ratio to the smallest extent, we get a FIT β with the ratio $|a_2^s/a_2^d| \approx 1.1$. By "the smallest extent," we mean that, if we continue to suppress the ratio down, the predicted branching ratios of some decay modes in Table 4 will be inconsistent with the experimental data. FIT A predicts an unusually large ratio $|a_2^V/a_1^V| \approx 1.1$, which indicates that the non-factorizable

Table 5. Predicted branching ratios for charmed mesons decaying to one pseudoscalar and one vector meson. Single prime and double primes are added to the representations to denote the singly Cabibbo-suppressed processes and doubly Cabibbo-suppressed processes

Decay modes		Representation	Experimental $\mathcal{B}(\times 10^{-2})$	Present $\mathcal{B}(\times 10^{-2})$ FIT A	Present $\mathcal{B}(\times 10^{-2})$ FIT B	LP[40] $\mathcal{B}(\times 10^{-2})$
	$K^{*-}\pi^+$	$T_V + E_P$	6.0 ± 0.5	5.93	5.97	4.656
	$K^-\rho^+$	$T_P + E_V$	10.2 ± 0.8	9.99	9.90	11.201
	$\overline{K}^{*0}\pi^0$	$\frac{1}{\sqrt{2}}(C_P - E_P)$	2.8 ± 0.4	2.72	2.81	3.208
	$\overline{K}^0\rho^0$	$\frac{1}{\sqrt{2}}(C_V - E_V)$	1.47 ± 0.29	1.49	1.25	0.759
	$\overline{K}^{*0}\eta$	$\frac{1}{\sqrt{3}}(C_P + E_P - E_V + SE_V)$	1.8 ± 0.4	1.50	1.94	
	$\overline{K}^0\omega$	$-\frac{1}{\sqrt{2}}(C_V + E_V)$	2.2 ± 0.4	2.11	1.80	1.855
	$\overline{K}^0\phi$	$-E_P - SE_P$	0.94 ± 0.11	0.95	0.90	
	K^+K^{*-}	$T'_V + E'_P$	0.20 ± 0.11	0.25	0.25	0.290
	K^-K^{*+}	$T'_P + E'_V$	0.38 ± 0.08	0.43	0.43	0.431
	$K^0\overline{K}^{*0}$	$E'_V - E'_P$	< 0.17	0.08	0.16	0.052
	$\overline{K}^0K^{*0}$	$E'_P - E'_V$	< 0.09	0.08	0.16	0.062
	$\pi^0\phi$	$\frac{1}{\sqrt{2}}(C'_P + SE'_P)$	< 0.14	0.12	0.12	0.105
	$\overline{K}^{*0}\eta'$	$-\frac{1}{\sqrt{6}}(C_P + E_P + 2E_V + 4SE_V)$	< 0.10	0.004	0.003	
D^0	$\eta\phi$	$\frac{1}{\sqrt{3}}(C'_P - 2SE'_P + SE'_V)$	< 2.8	0.035	0.034	
	$\pi^+\rho^-$	$-(T'_V + E'_P)$	–	0.34	0.35	0.485
	$\pi^-\rho^+$	$-(T'_P + E'_V)$	–	0.62	0.61	0.706
	$\pi^0\rho^0$	$\frac{1}{2}(C'_P + C'_V - E_P - E_V)$	–	0.19	0.16	0.216
	$\pi^0\omega$	$\frac{1}{2}(C'_V - C'_P + E'_P + E'_V + 2SE'_P)$	–	0.020	0.003	0.013
	$\eta\omega$	$-\frac{1}{\sqrt{6}}(C'_P + 2C'_V + SE'_V + 4SE'_P)$	–	0.13	0.10	
	$\eta'\omega$	$\frac{1}{2\sqrt{3}}(C'_P - C'_V + 4SE'_V - 2SE'_P)$	–	0.0007	0.0003	
	$\eta\rho^0$	$\frac{1}{\sqrt{6}}(2C'_V - C'_P - SE'_V)$	–	0.0039	0.0015	
	$\eta'\rho^0$	$\frac{1}{2\sqrt{3}}(C'_V + C'_P + 4SE'_V)$	–	0.012	0.009	0.039
	$K^{*+}\pi^-$	$-(T''_P + E''_V)$	–	0.029	0.029	0.025
	$K^+\rho^-$	$-(T''_V + E''_P)$	–	0.016	0.016	0.004
	$K^{*0}\pi^0$	$-\frac{1}{\sqrt{2}}(C''_P - E''_V)$	–	0.0052	0.0064	0.008
	$K^0\rho^0$	$-\frac{1}{\sqrt{2}}(C''_V - E''_P)$	–	0.0069	0.0059	
	$K^{*0}\eta$	$-\frac{1}{\sqrt{3}}(C''_P - E''_P + E''_V + SE''_V)$	–	0.0030	0.0041	
	$K^{*0}\eta'$	$\frac{1}{\sqrt{6}}(C''_P + 2E''_P + E''_V + 4SE''_V)$	–	0.0	0.0	
	$K^0\omega$	$\frac{1}{\sqrt{2}}(C''_V + E''_P)$	–	0.0076	0.0056	0.002
	$K^0\phi$	$E''_V + SE''_P$	–	0.0	0.0006	

contributions to a_2^V are of great importance. By constraining the value of $|a_2^V|$ to be as small as possible, we obtain FIT B with the ratio 0.9. The next leading order Wilson coefficients $c_1(m_c) = 1.174$ and $c_2(m_c) = -0.356$ in the naive dimensional regularization (NDR) scheme or $c_1(m_c) = 1.216$ and $c_2(m_c) = -0.424$ in the 't Hooft–Veltman (HV) scheme are given in [38] when $\Lambda_{\overline{MS}} = 0.215$ GeV. The present relatively large values of $|a_2^s|$, $|a_2^d|$, $|a_2^V|$ and $|a_2^P|$ cannot be explained from (21). They imply that non-factorizable contributions are of significance in both $D \to PP$ and $D \to PV$ decays. To fit the experimental result of $\mathrm{Br}(D^0 \to K^0\overline{K}^0) = (0.071 \pm 0.019)\%$, a_E^s should differ much from a_E^d. In $D \to PV$ decays, because we have considered the errors of experimental data in the χ^2 fit and used more experimental results as constraints, the present resulting parameters appear more reasonable than that of the case (I) solution presented in [15], as the strong phases of the parameters $a_1^{P,V}$ and $a_2^{P,V}$ are not in contradiction to that predicted from QCD.

We present the resultant predictions for a variety of charmed meson decay processes in Table 4 for $D \to PP$ decays and in Table 5 for $D \to PV$ decays. For comparison, we also list the results obtained in [40]. The predictions for a number of singly and doubly Cabibbo-suppressed modes can be used to test our present analysis in the near future.

Note that in the assumption of $SA_P = 0$, we have the branching ratio 14% for the process $D_s^+ \to \pi^+\omega$, which is much larger than the experimental result $(0.28 \pm 0.11)\%$. To accommodate the experimental data, a significant con-

Table 5. (continued)

	Decay modes	Representation	Experimental $\mathcal{B}(\times 10^{-2})$	Present $\mathcal{B}(\times 10^{-2})$ FIT A	Present $\mathcal{B}(\times 10^{-2})$ FIT B	LP [40] $\mathcal{B}(\times 10^{-2})$
	$\overline{K}^{*0}\pi^+$	$T_V + C_P$	1.92 ± 0.19	1.96	1.96	1.996
	$\pi^+\phi$	$C'_P - SA'_P$	0.61 ± 0.06	0.64	0.62	0.619
	$\overline{K}^0\rho^+$	$T_P + C_V$	6.6 ± 2.5	7.56	8.43	12.198
	$\pi^+\rho^0$	$-\frac{1}{\sqrt{2}}(T'_V + C'_P - A'_P + A'_V)$	0.104 ± 0.018	0.088	0.088	0.104
	$K^+\overline{K}^{*0}$	$T'_V - A'_V$	0.42 ± 0.05	0.44	0.44	0.436
	$\overline{K}^0 K^{*+}$	$T'_P - A'_P$	3.1 ± 1.4	1.43	1.25	1.515
	$K^+\rho^0$	$-\frac{1}{\sqrt{2}}(C''_V - A''_P)$	0.025 ± 0.012	0.030	0.025	0.029
	$K^{*0}\pi^+$	$-(C''_P + A''_V)$	0.036 ± 0.016	0.024	0.022	0.027
D^+	$K^+\phi$	$-(A''_V + SA''_P)$	< 0.013	0.0066	0.0067	
	$\pi^+\omega$	$\frac{1}{\sqrt{2}}(T'_V + C'_P + A'_V + A'_P + 2SA'_P)$	–	0.57	0.58	
	$\eta\rho^+$	$\frac{1}{\sqrt{3}}(T'_P + 2C'_V + A'_V + A'_P + SA'_V)$	–	0.24	0.43	
	$\eta'\rho^+$	$-\frac{1}{\sqrt{6}}(T'_P - C'_V + A'_V + A'_P + 4SA'_V)$	–	0.15	0.15	
	$\pi^0\rho^+$	$-\frac{1}{\sqrt{2}}(T'_P + C'_V + A'_P - A'_V)$	–	0.28	0.35	0.451
	$K^0\rho^+$	$-(C''_V + A''_P)$	–	0.025	0.022	0.042
	$\pi^0 K^{*+}$	$-\frac{1}{\sqrt{2}}(C''_P - A''_V)$	–	0.037	0.036	0.057
	$K^+\omega$	$-\frac{1}{\sqrt{2}}(C''_V + A''_P)$	–	0.012	0.011	
	$K^{*+}\eta$	$\frac{1}{\sqrt{3}}(T''_P - A''_P + A''_V + SA''_V)$	–	0.015	0.015	
	$K^{*+}\eta'$	$-\frac{1}{\sqrt{6}}(T''_P + 2A''_P + A''_V + 4SA''_V)$	–	0.00014	0.00016	
	$\overline{K}^{*0}K^+$	$C_P + A_V$	3.3 ± 0.9	3.34	3.42	4.812
	$\overline{K}^0 K^{*+}$	$C_V + A_P$	4.3 ± 1.4	4.98	4.66	2.467
	$\pi^+\rho^0$	$\frac{1}{\sqrt{2}}(A_V - A_P)$	$0.06^{\ddagger}(< 0.07)$	0.06	0.06	
	$\pi^+\phi$	$T_V + SA_P$	3.6 ± 0.9	3.08	2.93	4.552
	$\pi^+ K^{*0}$	$-(T'_V - A'_V)$	0.65 ± 0.28	0.33	0.35	0.445
	$K^+\rho^0$	$-\frac{1}{\sqrt{2}}(C'_P + A'_P)$	< 0.29	0.12	0.12	0.198
D_s^+	$K^+\phi$	$T'_V + C'_P + A'_V + SA'_P$	< 0.05	0.032	0.033	0.008
	$K^+\omega$	$-\frac{1}{\sqrt{2}}(C'_P - A'_P - 2SA'_P)$	–	0.40	0.39	0.178
	$K^0\rho^+$	$-(T'_P - A'_P)$	–	0.91	0.77	1.288
	$\pi^0 K^{*+}$	$-\frac{1}{\sqrt{2}}(C'_V + A'_V)$	–	0.13	0.13	0.076
	ηK^{*+}	$\frac{1}{\sqrt{3}}(T'_P + 2C'_V + A'_P - A'_V - SA'_V)$	–	0.038	0.047	0.146
	$\eta' K^{*+}$	$\frac{1}{\sqrt{3}}(2T'_P + C'_V + 2A'_P + A'_V + 4SA'_V)$	–	0.068	0.059	
	$K^{*0}K^+$	$-(T''_V + C''_P)$	–	0.0015	0.0015	0.006
	$K^{*+}K^0$	$-(T''_P + C''_P)$	–	0.0076	0.0085	0.018

‡ The central value of the E791 experiment [39].

tribution from the SA_P diagram, i.e. $SA_P \sim -A_p$, should be introduced [15].

5 SU(3) flavor symmetry breaking

As pointed out in [13,14], SU(3) breaking effects in charmed meson decays appear to be important. The violation may come from the finite strange quark mass, the final state interactions and resonances. In the SU(3) flavor symmetry limit, there are a number of relations among the different decay modes. Based on the above extracted values for the parameters, we can discuss how large are the SU(3) breaking effects in the $D \to PP$ and $D \to PV$ decays.

We present these relations in Table 6 for $D \to PP$ and Table 7 for $D \to PV$. The left hand side (LHS) values of the relations whose deviation from unit represents the breaking amounts of the SU(3) flavor symmetry relations are listed in the second columns.

It is noted that though these relations deviating from unit reflect the SU(3) flavor symmetry breaking effects, the ones composed of three decay modes and those com-

Table 6. SU(3) flavor symmetry relations of $D \to PP$ decay modes and breaking of the relations. $\lambda = V_{cs}^* V_{us}/V_{cs}^* V_{ud} \approx 0.226$. $\kappa = V_{cs}^* V_{us}/V_{cd}^* V_{us} \approx 4.446$

SU(3) symmetry relations	LHS of relations	
	FIT α	FIT β
$\frac{\mathcal{A}(D^0\to\pi^+\pi^-)+\sqrt{2}\mathcal{A}(D^0\to\pi^0\pi^0)}{\sqrt{2}\mathcal{A}(D^+\to\pi^+\pi^0)}=1$	1.00	1.00
$\frac{\mathcal{A}(D^0\to K^-\pi^+)+\sqrt{2}\mathcal{A}(D^0\to\overline{K}^0\pi^0)}{\mathcal{A}(D^+\to\overline{K}^0\pi^+)}=1$	1.00	1.00
$\frac{\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^0)+\kappa\mathcal{A}(D^+\to K^0\pi^+)}{\sqrt{2}\kappa\mathcal{A}(D^+\to K^+\pi^0)}=1$	0.49	0.79
$\frac{\lambda\mathcal{A}(D^+\to\overline{K}^0\pi^+)+\sqrt{2}\kappa\mathcal{A}(D^+\to K^+\pi^0)}{\kappa\mathcal{A}(D^+\to K^0\pi^+)}=1$	1.56	1.11
$\frac{\sqrt{2}\kappa\mathcal{A}(D^+\to\pi^0K^+)-\kappa\mathcal{A}(D^+\to K^0\pi^+)}{\lambda\mathcal{A}(D^+\to\overline{K}^0\pi^+)}=1$	2.21	1.82
$\frac{\mathcal{A}(D^0\to K^+K^-)}{\kappa\mathcal{A}(D^0\to K^+\pi^-)}=1$	1.27	1.24
$\frac{\kappa\mathcal{A}(D^0\to K^+\pi^-)}{\mathcal{A}(D^0\to\pi^+\pi^-)}=1$	1.43	1.43
$\frac{\kappa\mathcal{A}(D^0\to K^+\pi^-)}{\lambda\mathcal{A}(D^0\to K^-\pi^+)}=1$	1.20	1.24
$\frac{\lambda\mathcal{A}(D^0\to\overline{K}^0\pi^0)}{\mathcal{A}(D^0\to\pi^0\pi^0)}=1$	1.26	1.12
$\frac{\lambda\mathcal{A}(D^0\to\overline{K}^0\pi^0)}{\kappa\mathcal{A}(D^0\to K^0\pi^0)}=1$	1.78	1.10
$\frac{\mathcal{A}(D^+\to K^+\overline{K}^0)}{\sqrt{2}\kappa\mathcal{A}(D^+\to K^+\pi^0)}=1$	0.89	0.86
$\frac{\sqrt{2}\kappa\mathcal{A}(D^+\to K^+\pi^0)}{\mathcal{A}(D_s^+\to K^0\pi^+)}=1$	1.24	1.24
$\frac{\lambda\mathcal{A}(D_s^+\to\overline{K}^0K^+)}{\sqrt{2}\mathcal{A}(D_s^+\to K^+\pi^0)}=1$	1.34	0.98
$\frac{\kappa\mathcal{A}(D^+\to K^0\pi^+)}{\lambda\mathcal{A}(D_s^+\to K^+\pi^0)}=1$	1.08	1.14
$\frac{\lambda\mathcal{A}(D^+\to\overline{K}^0\pi^+)}{\sqrt{2}\mathcal{A}(D^+\to\pi^0\pi^+)}=1$	0.55	0.67

posed of two decay modes have different sources of breaking terms. To be clear, we take the expressions

$$\frac{|\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})+\sqrt{2}\mathcal{A}(D^+\to\pi^+\rho^0)|}{|\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)|}$$

and

$$\frac{|\mathcal{A}(D^0\to K^+K^{*-})|}{|\mathcal{A}(D^0\to\pi^+\rho^-)|}$$

as examples. We have

$$\begin{aligned}
&\frac{|\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})+\sqrt{2}\mathcal{A}(D^+\to\pi^+\rho^0)|}{|\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)|} \qquad (41)\\
&=|(T_V+C_P)(D^+\to\pi^+\overline{K}^{*0})\\
&\qquad -(T_V+C_P-A_P+A_V)(D^+\to\pi^+\rho^0)|\\
&\quad /|A_V(D_s^+\to\pi^+\rho^0)-A_P(D_s^+\to\pi^+\rho^0)|,\\
&\frac{|\mathcal{A}(D^0\to K^+K^{*-})|}{|\mathcal{A}(D^0\to\pi^+\rho^-)|} \qquad (42)\\
&=|T_V(D^0\to K^+K^{*-})+E_P(D^0\to K^+K^{*-})|\\
&\quad /|-T_V(D^0\to\pi^+\rho^-)-E_P(D^0\to\pi^+\rho^-)|.
\end{aligned}$$

In the limit of SU(3) flavor symmetry, we have the following relations:

$$T_V(D^+\to\pi^+\overline{K}^{*0})=T_V(D^+\to\pi^+\rho^0), \qquad (43)$$

$$C_P(D^+\to\pi^+\overline{K}^{*0})=C_P(D^+\to\pi^+\rho^0), \qquad (44)$$

$$A_V(D^+\to\pi^+\rho^0)=A_V(D_s^+\to\pi^+\rho^0), \qquad (45)$$

$$A_P(D^+\to\pi^+\rho^0)=A_P(D_s^+\to\pi^+\rho^0), \qquad (46)$$

$$T_V(D^0\to K^+K^{*-})=T_V(D^0\to\pi^+\rho^-), \qquad (47)$$

$$E_P(D^0\to K^+K^{*-})=E_P(D^0\to\pi^+\rho^-), \qquad (48)$$

which make the ratios (41) and (42) equal to one. But from (8)–(13), one can find that relations in (43)–(48) are in general not valid. The different masses of the charmed mesons and the final light mesons, and the different values of formfactors and decay constants can break the relations in (43)–(48), and thus break the SU(3) flavor symmetry relations in (41) and (42). In addition, by comparing with (41) and (42), one can see that the relations concerning only two decay modes represent the relative SU(3) flavor symmetry breaking amounts of the same diagrams which we call the main diagrams for convenience in later use, while the relations consisting of three decay modes contain additional SU(3) flavor symmetry breaking effects from the other diagrams. So in the relations containing three decay modes, if the SU(3) flavor symmetry breaking contributions of the other diagrams have comparable amounts in comparison with the main diagrams, then the relations will be broken down badly. The main diagrams $|T+C|$ in $D^+\to\pi^+\overline{K}^0$, $|A_V-A_P|$ in $D_s^+\to\pi^+\rho^0$ and $|T_V+C_P|$ in $D^+\to\pi^+\overline{K}^{*0}$ are relatively small, which usually leads

Table 7. SU(3) flavor symmetry relations of $D \to PV$ decays and breaking of the relations. $\lambda = V_{cs}^* V_{us}/V_{cs}^* V_{ud} \approx 0.226$. $\kappa = V_{cs}^* V_{us}/V_{cd}^* V_{us} \approx 4.446$

SU(3) symmetry relations	LHS of relations	
	FIT A	FIT B
$\frac{\mathcal{A}(D^0\to\pi^+K^{*-})+\sqrt{2}\mathcal{A}(D^0\to\pi^0\overline{K}^{*0})}{\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})}=1$	1.00	1.00
$\frac{\mathcal{A}(D^0\to\rho^+K^{-})+\sqrt{2}\mathcal{A}(D^0\to\rho^0\overline{K}^{0})}{\mathcal{A}(D^+\to\overline{K}^{0}\rho^+)}=1$	1.00	1.00
$\frac{\mathcal{A}(D^0\to\overline{K}^{0}\phi)-\mathcal{A}(D_s^+\to\pi^+\phi)}{\mathcal{A}(D^0\to\pi^+K^{*-})}=1$	1.00	1.00
$\frac{\mathcal{A}(D^0\to\pi^+K^{*-})+\mathcal{A}(D^0\to\overline{K}^{0}\phi)}{\mathcal{A}(D_s^+\to\pi^+\phi)}=1$	0.99	0.99
$\frac{\mathcal{A}(D^0\to\pi^+K^{*-})-\mathcal{A}(D_s^+\to\pi^+\phi)}{\mathcal{A}(D^0\to\overline{K}^{0}\phi)}=1$	1.00	1.00
$\frac{\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)+\sqrt{2}\mathcal{A}(D^+\to\pi^+\rho^0)}{\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})}=1$	0.88	0.88
$\frac{\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})+\sqrt{2}\mathcal{A}(D^+\to\pi^+\rho^0)}{\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)}=1$	0.60	0.59
$\frac{\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})+\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)}{\sqrt{2}\mathcal{A}(D^+\to\pi^+\rho^0)}=1$	1.10	1.10
$\frac{\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)-\sqrt{2}\mathcal{A}(D^+\to\pi^0\rho^+)}{\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^{0})}=1$	1.03	1.03
$\frac{\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^{0})+\sqrt{2}\mathcal{A}(D^+\to\pi^0\rho^+)}{\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)}=1$	1.48	1.10
$\frac{\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)-\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^{0})}{\sqrt{2}\mathcal{A}(D^+\to\pi^0\rho^+)}=1$	0.95	0.97
$\frac{\lambda\mathcal{A}(D_s^+\to K^+\overline{K}^{*0})+\mathcal{A}(D^+\to K^+\overline{K}^{*0})}{\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})}=1$	0.58	0.57
$\frac{\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})-\mathcal{A}(D^+\to K^+\overline{K}^{*0})}{\lambda\mathcal{A}(D_s^+\to K^+\overline{K}^{*0})}=1$	1.17	1.16
$\frac{\lambda\mathcal{A}(D_s^+\to K^+\overline{K}^{*0})-\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})}{\mathcal{A}(D^+\to K^+\overline{K}^{*0})}=1$	0.96	0.97
$\frac{\lambda\mathcal{A}(D_s^+\to\overline{K}^{0}K^{*+})+\mathcal{A}(D^+\to\overline{K}^{0}K^{*+})}{\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^{0})}=1$	1.09	1.19
$\frac{\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^{0})-\mathcal{A}(D^+\to\overline{K}^{0}K^{*+})}{\lambda\mathcal{A}(D_s^+\to\overline{K}^{0}K^{*+})}=1$	1.01	0.97
$\frac{\lambda\mathcal{A}(D_s^+\to\overline{K}^{0}K^{*+})-\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^{0})}{\mathcal{A}(D^+\to\overline{K}^{0}K^{*+})}=1$	0.98	0.96
$\frac{\lambda\mathcal{A}(D_s^+\to\overline{K}^{0}K^{*+})+\sqrt{2}\mathcal{A}(D_s^+\to K^{*+}\pi^0)}{\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)}=1$	1.42	1.31
$\frac{\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)+\sqrt{2}\mathcal{A}(D_s^+\to K^{*+}\pi^0)}{\lambda\mathcal{A}(D_s^+\to\overline{K}^{0}K^{*+})}=1$	0.92	0.95
$\frac{\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)+\lambda\mathcal{A}(D_s^+\to\overline{K}^{0}K^{*+})}{\sqrt{2}\mathcal{A}(D_s^+\to K^{*+}\pi^0)}=1$	1.11	1.07
$\frac{\mathcal{A}(D_s^+\to K^0\rho^+)}{\mathcal{A}(D^+\to\overline{K}^{0}K^{*+})}=1$	0.91	0.89
$\frac{\mathcal{A}(D_s^+\to\pi^+K^{*0})}{\mathcal{A}(D^+\to K^+\overline{K}^{*0})}=1$	0.93	0.94
$\frac{\mathcal{A}(D^0\to K^+K^{*-})}{\kappa\mathcal{A}(D^0\to K^+\rho^-)}=1$	1.05	1.05
$\frac{\lambda\mathcal{A}(D^0\to\pi^+K^{*-})}{\mathcal{A}(D^0\to\pi^+\rho^-)}=1$	1.05	1.05
$\frac{\mathcal{A}(D^0\to K^+K^{*-})}{\lambda\mathcal{A}(D^0\to\pi^+K^{*-})}=1$	1.14	1.14
$\frac{\mathcal{A}(D^0\to K^-K^{*+})}{\kappa\mathcal{A}(D^0\to\pi^-K^{*+})}=1$	1.08	1.08
$\frac{\lambda\mathcal{A}(D^0\to K^-\rho^+)}{\mathcal{A}(D^0\to\pi^-\rho^+)}=1$	1.09	1.09
$\frac{\mathcal{A}(D^0\to K^-K^{*+})}{\lambda\mathcal{A}(D^0\to K^-\rho^+)}=1$	1.08	1.08

to a significant breaking for the relations when taking $A(D^+ \to \pi^+ \overline{K}^0)$, $A(D_s^+ \to \pi^+ \rho^0)$ and $A(D^+ \to \pi^+ \overline{K}^{*0})$ as denominators. We present explicitly some of these relations calculated from the parameters of FIT α and FIT A as follows:

$$\frac{|\sqrt{2}\kappa \mathcal{A}(D^+ \to K^+ \pi^0) - \kappa \mathcal{A}(D^+ \to K^0 \pi^+)|}{|\lambda \mathcal{A}(D^+ \to \overline{K}^0 \pi^+)|} = 2.21, \tag{49}$$

$$\frac{|\lambda \mathcal{A}(D^+ \to \pi^+ \overline{K}^{*0}) + \sqrt{2}\mathcal{A}(D^+ \to \pi^+ \rho^0)|}{|\lambda \sqrt{2}\mathcal{A}(D_s^+ \to \pi^+ \rho^0)|} = 0.60, \tag{50}$$

$$\frac{|\lambda \mathcal{A}(D_s^+ \to \overline{K}^0 K^{*+}) + \sqrt{2}\mathcal{A}(D_s^+ \to K^{*+} \pi^0)|}{|\lambda \sqrt{2}\mathcal{A}(D_s^+ \to \pi^+ \rho^0)|} = 1.42, \tag{51}$$

$$\frac{|\lambda \mathcal{A}(D_s^+ \to K^+ \overline{K}^{*0}) + \mathcal{A}(D^+ \to K^+ \overline{K}^{*0})|}{|\lambda \mathcal{A}(D^+ \to \pi^+ \overline{K}^{*0})|} = 0.58. \tag{52}$$

It is obvious that the SU(3) flavor symmetry analysis is not applicable to such processes.

Besides the mass factors, the formfactors and decay constants, one should also consider the contributions of a_i factors when studying the SU(3) symmetry breaking effects in $D \to PP$ decay modes. The situations are more complicated than that in $D \to PV$ decay modes. General speaking, the SU(3) flavor symmetry breaking effects are more important in $D \to PP$ decays. The first two relations in Tables 6 and 7 still are conserved because all the decay modes in them form an isospin triangle respectively.

6 Summary and conclusion

We have performed a χ^2 fitting analysis on the $D \to PP$ and $D \to PV$ decays in the formalism of the factorization hypotheses. To fit the experimental data, it is vital to consider the SU(3) flavor symmetry breaking effects of the coefficients a_is in the $D \to PP$ decay modes. In $D \to PV$ decays, the final state hadron structure of the pseudoscalar and vector mesons has a more important impact on the coefficients a_is than the SU(3) symmetry breaking effects. The non-factorizable contributions, as well as that of the exchange and annihilation diagrams, are found to be important in these decays. In the formalism of the relations obtained in the SU(3) symmetry limit, the total SU(3) symmetry breaking amount of certain processes in $D \to PP$ can reach 120% when the three symmetry breaking effects due to a_i factors, mass factors and due to formfactors and decay constants are to be coherently added. The total breaking amount of some processes in $D \to PV$ can add up to 50%. The breaking amount of the SU(3) symmetry relations in some channels is so significant that it becomes unreliable to use the SU(3) relations to make predictions for some decay modes. More precise measurements on the process $D^+ \to \overline{K}^0 K^{*+}$ are important for understanding the SU(3) symmetry breaking effects and non-factorizable contributions. As an independent check, it is useful to measure the process $D_s^+ \to K^0 \rho^+$. The processes $D^0 \to \pi^+ \rho^-$, $D^0 \to \pi^- \rho^+$, $D^0 \to \pi^0 \rho^0$, $D^+ \to \pi^+ \omega$, $D^+ \to \pi^0 \rho^+$, $D^+ \to K^0 \rho^+$, $D^+ \to \pi^0 K^{*+}$, $D_s^+ \to K^+ \omega$ and $D_s^+ \to \pi^0 K^{*+}$ are predicted to be at the experimental sensitivity. It is expected that one may explore the final hadron structure and SU(3) flavor symmetry breaking effects in $D \to PP$ and $D \to PV$ decays in BES, CLEO-c, BaBar and Belle.

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References

1. L.L. Chau, H.Y. Cheng, Phys. Rev. Lett. **56**, 1655 (1986); Phys. Rev. D **36**, 137 (1987); Phys. Lett. B **222**, 285 (1989); Mod. Phys. Lett. A **4**, 877 (1989); L.L. Chau, Phys. Rep. **95**, 1 (1983)
2. M. Bauer, B. Stech, M. Wirbel, Z. Phys. C **34**, 103 (1987)
3. V. Barger, S. Pakvasa, Phys. Rev. Lett. **43**, 812 (1979); H.J. Lipkin, Phys. Rev. Lett. **44**, 710 (1980); X.Y. Pham, Phys. Lett. B **193**, 331 (1987); J.G. Korner, K. Schilcher, M. Wirbel, Y.L. Wu, Z. Phys. C **48**, 663 (1990); F. Buccella, M. Lusignoli, G. Miele, A. Pugliese, P. Santorelli, Phys. Rev. D **51**, 3478 (1995); F. Buccella, M. Lusignoli, A. Pugliese, Phys. Lett. B **379**, 249 (1996); M. Ablikim, D.S. Du, M.Z. Yang, Phys. Lett. B **536**, 34 (2002)
4. S. Bianco, F.L. Fabbri, D. Benson, I. Bigi, hep-ex/0309021
5. S.E. Csorna et al., CLEO Collaboration, Phys. Rev. D **65**, 092001 (2002); B.D. Yabsley, hep-ex/0311057; A.A. Petrov, hep-ph/0403030
6. X.Q. Li, B.S. Zou, Phys. Lett. B **399**, 297 (1997); Y.S. Dai, D.S. Du, X.Q. Li, Z.T. Wei, B.S. Zou, Phys. Rev. D **60**, 014014 (1999); S. Fajfer, J. Zupan, Int. J. Mod. Phys. A **14**, 4161 (1999); P. Zenczykowski, Phys. Lett. B **460**, 390 (1999); J.O. Eeg, S. Fajfer, J. Zupan, Phys. Rev. D **64**, 034010 (2001); J.W. Li, M.Z. Yang, D.S. Du, hep-ph/0206154; M. Ablikim, D.S. Du, M.Z. Yang, hep-ph/0211413; S. Fajfer, A. Prapotnik, P. Singer, J. Zupan, Phys. Rev. D **68**, 094012 (2003)
7. M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999)
8. H.N. Li, H.L. Yu, Phys. Rev. Lett. **74**, 4388 (1995)
9. A.N. Kamal, R.C. Verma, Phys. Rev. D **35**, 3515 (1987); Erratum D **36**, 3527 (1987); R.C. Verma, A.N. Kamal, Phys. Rev. D **43**, 829 (1991)
10. J.L. Rosner, Phys. Rev. D **60**, 114026 (1999)
11. C.W. Chiang, Z. Luo, J.L. Rosner, Phys. Rev. D **67**, 014001 (2003)
12. M. Gronau, O.F. Hernández, D. London, J.L. Rosner, Phys. Rev. D **50**, 4529 (1994); **52**, 6356, 6374 (1995)
13. L.L. Chau, H.Y. Cheng, Phys. Lett. B **333**, 514 (1994)
14. M. Gronau, D. Pirjol, Phys. Rev. D **62**, 077301 (2000)
15. M. Zhong, Y.L. Wu, W.Y. Wang, Eur. Phys. J. C (2003),
16. F.E. Close, H.J. Lipkin, Phys. Lett. B **551**, 337 (2003)

17. M. Gronau, J.L. Rosner, Phys. Rev. D **53**, 2516 (1996); A.S. Dighe, M. Gronau, J.L. Rosner, Phys. Lett. B **367**, 357 (1996); **377**, 325(E) (1996)
18. T. Feldmann, P. Kroll, Eur. Phys. J. C **5**, 327 (1998); T. Feldmann, P. Kroll, B. Stech, Phys. Rev. D **58**, 114006 (1998); Phys. Lett. B **449**, 339 (1999); T. Feldmann, P. Kroll, Phys. Scripta T **99**, 13 (2002)
19. L.L. Chau, H.Y. Cheng, Phys. Rev. D **39**, 2788 (1989); L.L. Chau, H.Y. Cheng, T. Huang, Z. Phys. C **53**, 413 (1992); H.Y. Cheng, B. Tseng, Phys. Rev. D **59**, 014034 (1999)
20. M. Neubert, V. Riekert, Q.P. Xu, B. Stech, in Heavy Flavors, edited by A.J. Buras, H. Lindner (World Scientific, Singapore 1992); H.Y. Cheng, Phys. Lett. B **335**, 428 (1994)
21. H.Y. Cheng, Eur. Phys. J. C **26**, 551 (2003)
22. P. Zenczykowski, Acta Phys. Polon. B **28**, 1605 (1997)
23. A. Khodjamirian, R. Rückl, Adv. Ser. Direct High Energy Phys. **15**, 345 (1998)
24. W.Y. Wang, Y.L. Wu, Phys. Lett. B **515**, 57 (2001); **519**, 219 (2001); M. Zhong, Y.L. Wu, W.Y. Wang, Int. J. Mod. Phys A **18**, 1959 (2003); W.Y. Wang, Y.L. Wu, M. Zhong, J. Phys. G **29**, 2743 (2003)
25. J.M. Flynn, C.T. Sachrajda, Adv. Ser. Direct. High Energy Phys. **15**, 402 (1998)
26. A. Abada, D. Becirevic, Ph. Boucaud, J.P. Leroy, V. Lubicz, F. Mescia, Nucl. Phys. B **619**, 565 (2001)
27. D. Scora, N. Isgur, Phys. Rev. D **52**, 2783 (1995)
28. D. Melikhov, Phys. Rev. D **53**, 2460 (1996); **56**, 7089 (1997)
29. M. Wirbel, B. Stech, M. Bauer, Z. Phys. C **29**, 637 (1985)
30. D. Melikhov, B. Stech, Phys. Rev. D **62**, 014006 (2000)
31. S. Gusken et al., Nucl. Phys. (Proc. Suppl.) **47**, 485 (1996)
32. J.M. Flynn, C.T. Sachrajda, Adv. Ser. Direct. High Energy Phys. **15**, 402 (1998)
33. P. Ball, V. Braun, H. Dosch, Phys. Lett. B **273**, 316 (1991); Phys. Rev. D **44**, 3567 (1991); P. Ball, Phys. Rev. D **48**, 3190 (1993)
34. W.Y. Wang, Y.L. Wu, M. Zhong, Phys. Rev. D **67**, 014024 (2003)
35. A. Ali, G. Kramer, Cai-Dian Lü, Phys. Rev. D **58**, 094009 (1998)
36. M. Neubert, B. Stech, Adv. Ser. Direct. High Energy Phys. **15**, 294 (1998)
37. K. Hagiwara et al., Phys. Rev. D **66**, 010001 (2002) (http://pdg.lbl.gov)
38. G. Buchalla, A.J. Buras, M.E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996)
39. Fermilab E791 Collabortion, E.M. Aitala et al., Phys. Rev. Lett. **86**, 765 (2001)
40. M. Lusignoli, A. Pugliese, hep-ph/0210071